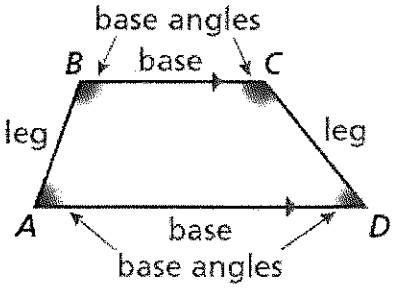
Trapezoids and Kites

Vocabulary:

Trapezoid

- a quadrilateral with exactly one pair of parallel sides
- the parallel sides are the **bases**
- the **base angles** of a trapezoid are two consecutive angles whose common side is a base
- a trapezoid has two pairs of base angles

In trapezoid ABCD, $\angle A$ and $\angle D$ are one pair of base angles, and $\angle B$ and $\angle C$ are the second pair.



The nonparallel sides are the **legs** of the trapezoid.

If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**

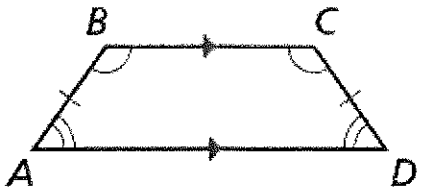
Isosceles trapezoid

Isosceles Trapezoid Base Angles Theorem

- if a trapezoid is isosceles, then each pair of base angles is congruent

if trapezoid ABCD is isosceles

then $\angle A \cong \angle D$ and $\angle B \cong \angle C$

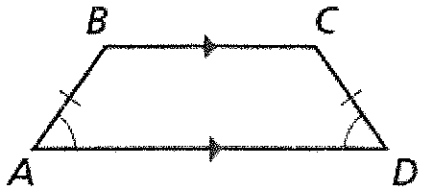


Isosceles Trapezoid Base Angles Converse

- if a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid

if $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$)

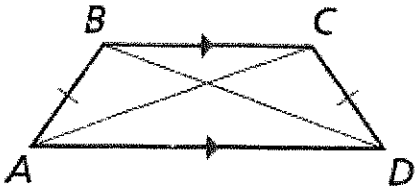
then trapezoid ABCD is isosceles



Isosceles Trapezoid Diagonals Theorem

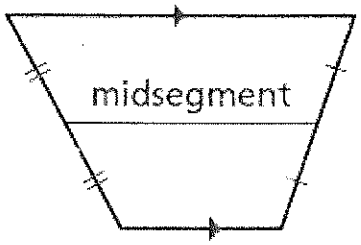
- a trapezoid is isosceles if and only if its diagonals are congruent

Trapezoid ABCD is isosceles if and only if $\overline{AC} \cong \overline{BD}$



Midsegment of a Trapezoid

- the segment that connects the midpoints of its legs

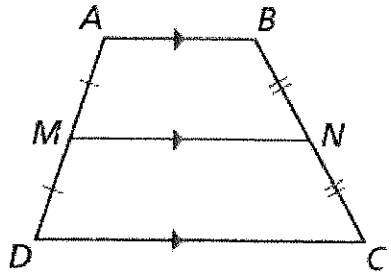


Trapezoid Midsegment Theorem

- the midsegment of a trapezoid is parallel to each base
- its length is one-half the sum of the lengths of the bases

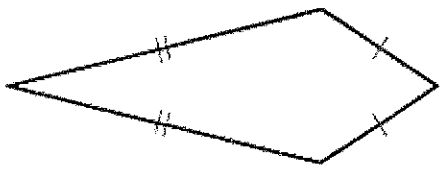
if $\overline{MN}$ is the midsegment of trapezoid ABCD

then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = 1/2(AB + CD)$



Kite

- a quadrilateral that has two pairs of consecutive congruent sides
- opposite sides are not congruent
- the congruent angles of a kite are formed by the noncongruent adjacent sides.

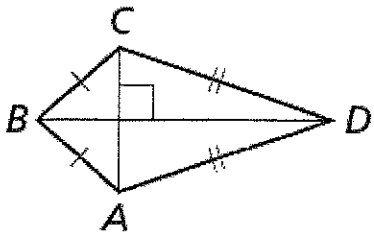


Kite Diagonals Theorem

- if a quadrilateral is a kite, then its diagonals are perpendicular

if quadrilateral ABCD is a kite

then $\overline{AC} \perp \overline{BD}$

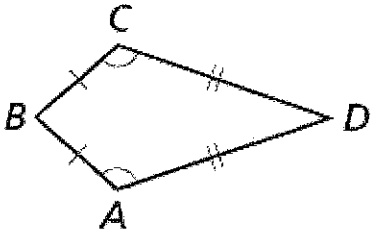


Kite Opposite Angles Theorem

- if a quadrilateral is a kite, then exactly one pair of opposite angles are congruent

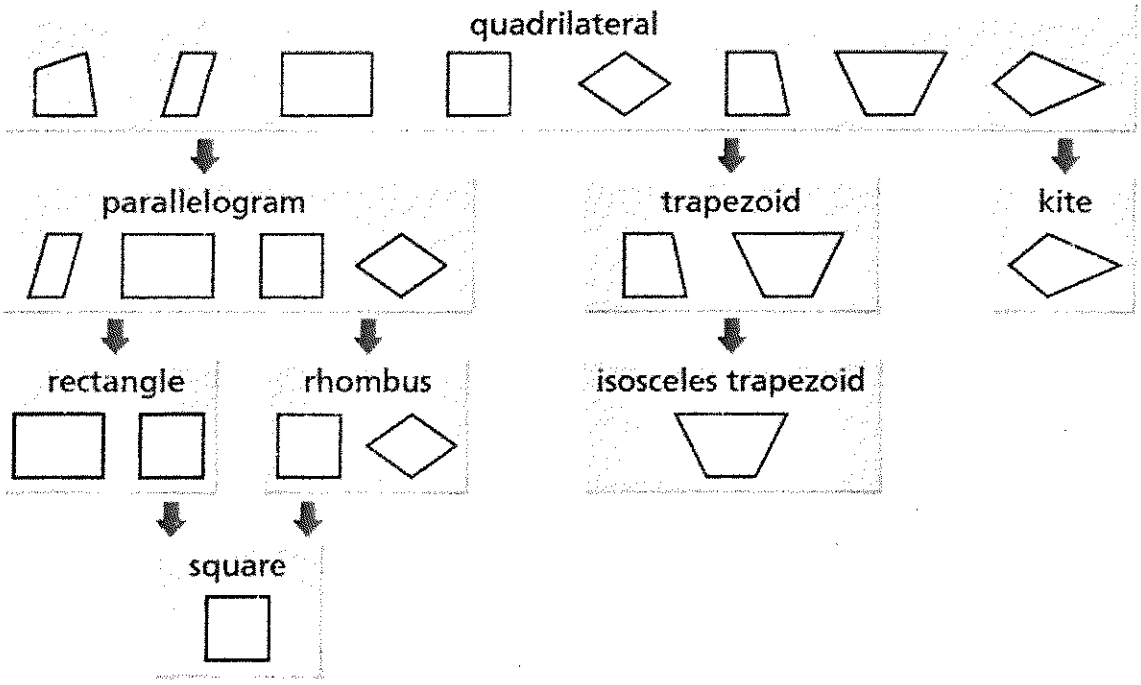
if quadrilateral ABCD is a kite and $\overline{BC} \cong \overline{BA}$

then $\angle A \cong \angle C$ and $\angle B \ncong \angle D$



Identifying Special Quadrilaterals

Each shape in the diagram has the properties of the shapes linked above it. For example, a rhombus has the properties of a parallelogram and a quadrilateral.



Example 1:

Prove that ABCD is a trapezoid and decide whether it is isosceles. Show all work.

$Slope_{AB}$ = _____

$Slope_{BC}$ = _____

$Slope_{CD}$ = _____

$Slope_{AD}$ = _____

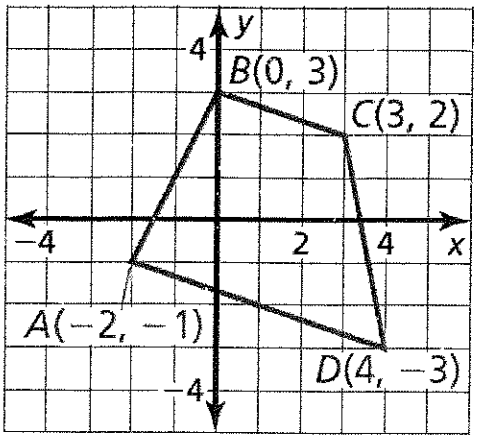
AB = _____

BC = _____

CD = _____

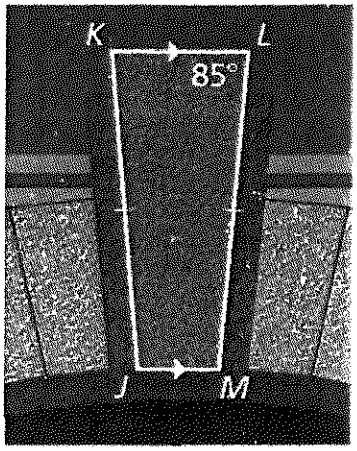
AD = _____

Isosceles? Yes/No



Example 2: Using Properties of Isosceles Trapezoids

The stone above the arch in the diagram is an isosceles trapezoid. Find $m\angle K$, $m\angle M$, and $m\angle J$. Explain your reasoning.



$m\angle J =$ _____

$m\angle K =$ _____

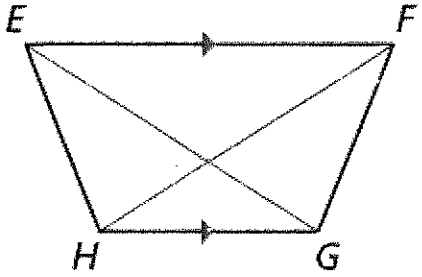
$m\angle M =$ _____

Reasoning:

Example 3:

If EG = FH, is trapezoid EFGH isosceles? Yes/No

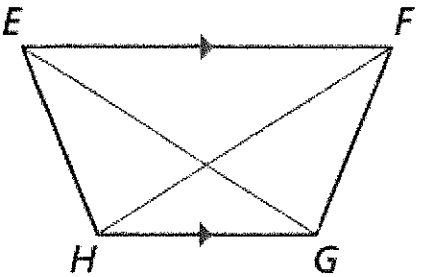
Explain your reasoning:



Example 4:

If $m\angle HEF = 70°$ and $m\angle FGH = 110°$, is trapezoid EFGH isosceles? Yes/No

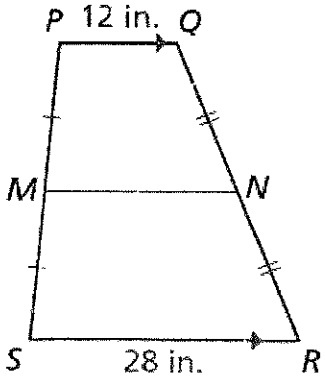
Explain your reasoning:



Example 5: Using the Midsegment of a Trapezoid

In the diagram, $\overline{MN}$ is the midsegment of trapezoid PQRS. Find MN. Show all work.

MN = _____

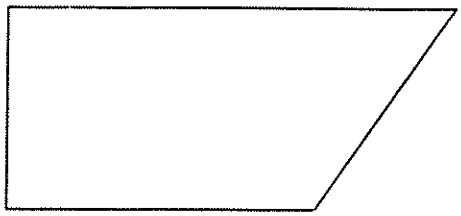


Example 6:

In trapezoid JKLM, ∠J and ∠M are right angles, and JK = 9 centimeters. The length of midsegment $\overline{NP}$ of trapezoid JKLM is 12 centimeters.

Using a ruler, label trapezoid JKLM and sketch its midsegment.
Find ML. Explain your reasoning.

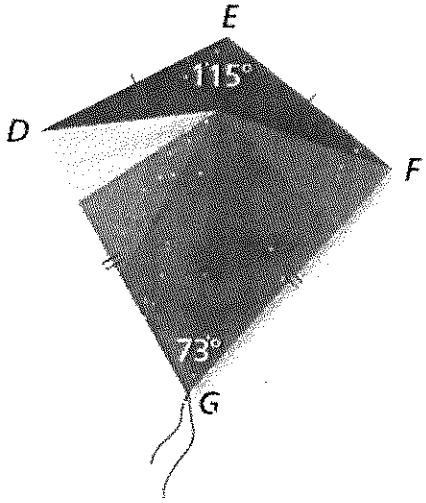
ML = _____



Example 7: Finding Angle Measures in a Kite

Find m∠D in the kite shown. Show all work.

m∠D = _____



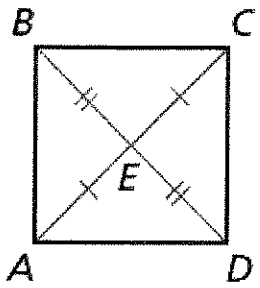
Example 8:

In a kite, the measures of the angles are $3x°$, $75°$, $90°$, and $120°$. Find the value of x. What are the measures of the angles that are congruent? Show all work.

$x =$ _____

Congruent angle measures = _____

Example 9: Identifying a Quadrilateral



What is the most specific name for quadrilateral ABCD? Explain your reasoning.

Hint: ABCD looks like a square. But you must rely **only** on marked information when you interpret a diagram.

Quadrilateral: _____

Reasoning:

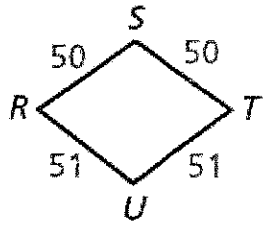
Example 10:

Quadrilateral DEFG has at least one pair of opposite sides congruent. What types of quadrilaterals meet this condition?

Example 11:

Give the most specific name for each quadrilateral. Explain your reasoning.

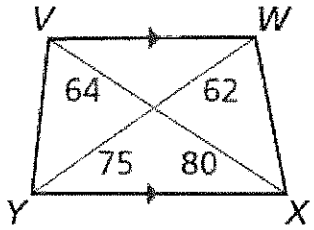
a.



_____

Reasoning:

b.



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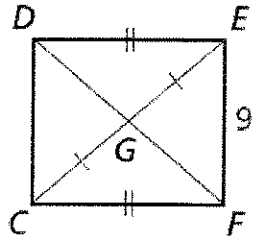
Reasoning:

c.



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Reasoning: