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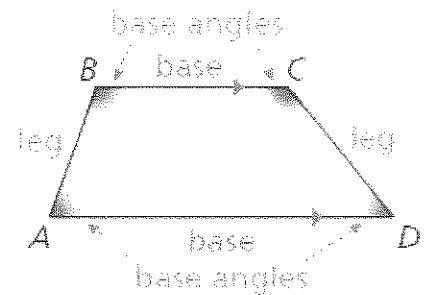
### Trapezoids and Kites

#### Vocabulary:

##### Trapezoid

- a quadrilateral with exactly one pair of parallel sides
- the parallel sides are the **bases**
- the **base angles** of a trapezoid are two consecutive angles whose common side is a base
- a trapezoid has two pairs of base angles

In trapezoid ABCD,  $\angle A$  and  $\angle D$  are one pair of base angles, and  $\angle B$  and  $\angle C$  are the second pair.



The nonparallel sides are the **legs** of the trapezoid.

If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**

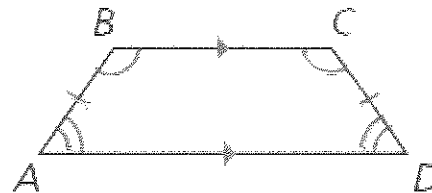


#### Isosceles Trapezoid Base Angles Theorem

- if a trapezoid is isosceles, then each pair of base angles is congruent

if trapezoid ABCD is isosceles

then  $\angle A \cong \angle D$  and  $\angle B \cong \angle C$

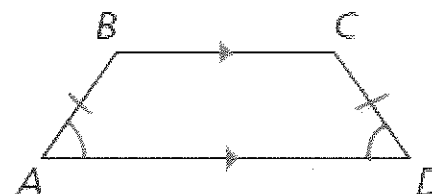


#### Isosceles Trapezoid Base Angles Converse

- if a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid

if  $\angle A \cong \angle D$  (or if  $\angle B \cong \angle C$ )

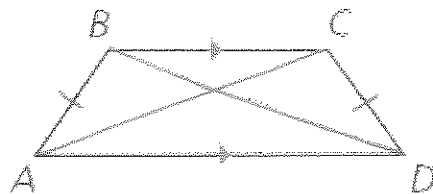
then trapezoid ABCD is isosceles



**Isosceles Trapezoid Diagonals Theorem**

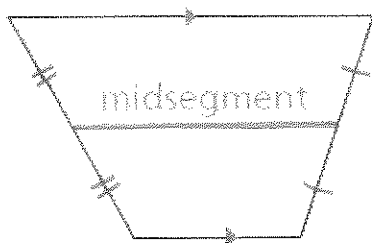
- a trapezoid is isosceles if and only if its diagonals are congruent

Trapezoid ABCD is isosceles if and only if  $\overline{AC} \cong \overline{BD}$



**Midsegment of a Trapezoid**

- the segment that connects the midpoints of its legs

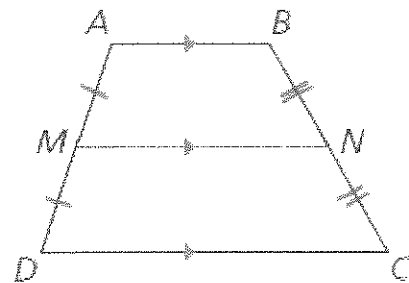


**Trapezoid Midsegment Theorem**

- the midsegment of a trapezoid is parallel to each base
- its length is one-half the sum of the lengths of the bases

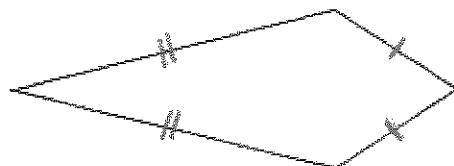
if  $\overline{MN}$  is the midsegment of trapezoid ABCD

then  $\overline{MN} \parallel \overline{AB}$ ,  $\overline{MN} \parallel \overline{DC}$ , and  $MN = 1/2(AB + CD)$



**Kite**

- a quadrilateral that has two pairs of consecutive congruent sides
- opposite sides are not congruent
- the congruent angles of a kite are formed by the noncongruent adjacent sides.

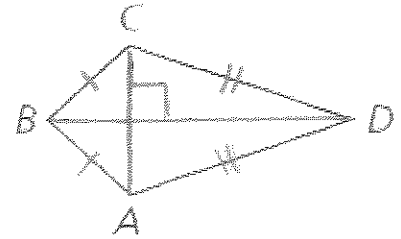


**Kite Diagonals Theorem**

- if a quadrilateral is a kite, then its diagonals are perpendicular

if quadrilateral ABCD is a kite

then  $\overline{AC} \perp \overline{BD}$

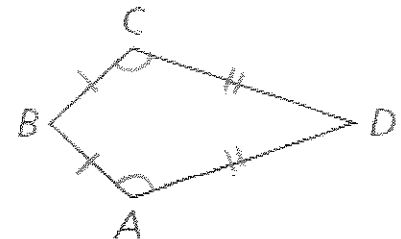


**Kite Opposite Angles Theorem**

- if a quadrilateral is a kite, then exactly one pair of opposite angles are congruent

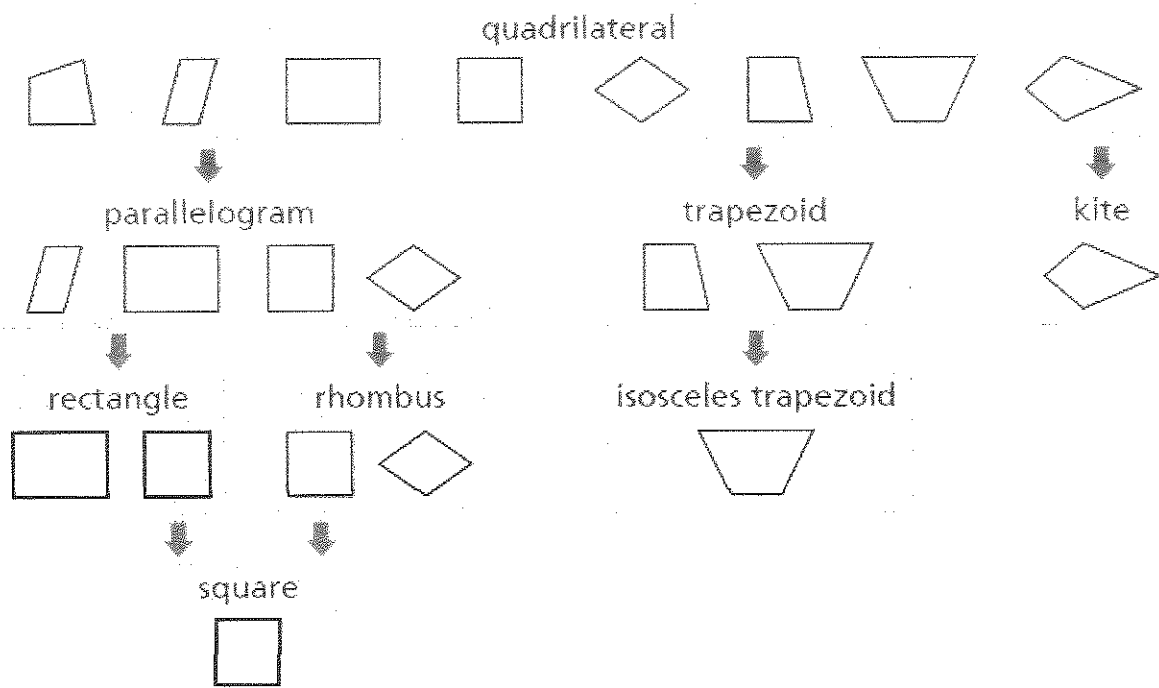
if quadrilateral ABCD is a kite and  $\overline{BC} \cong \overline{BA}$

then  $\angle A \cong \angle C$  and  $\angle B \cong \angle D$



**Identifying Special Quadrilaterals**

Each shape in the diagram has the properties of the shapes linked above it. For example, a rhombus has the properties of a parallelogram and a quadrilateral.



Example 1:

Prove that ABCD is a trapezoid and decide whether it is isosceles. Show all work.

Slope<sub>AB</sub> =  $\frac{2}{4}$  =  $\frac{1}{2}$

Slope<sub>BC</sub> =  $-\frac{1}{3}$

Slope<sub>CD</sub> =  $-\frac{5}{2}$

Slope<sub>AD</sub> =  $-\frac{1}{3}$  =  $-\frac{2}{6}$

BC || AD

AB =  $\sqrt{20}$

BC =  $\sqrt{10}$

CD =  $\sqrt{26}$

AD =  $\sqrt{40}$

Isosceles? Yes/No

$(AB)^2 = 4^2 + 2^2$

$(AB)^2 = 16 + 4$

$AB = \sqrt{20}$

$(CD)^2 = 5^2 + 1^2$

$(CD)^2 = 25 + 1$

$CD = \sqrt{26}$

$(BC)^2 = 3^2 + 1^2$

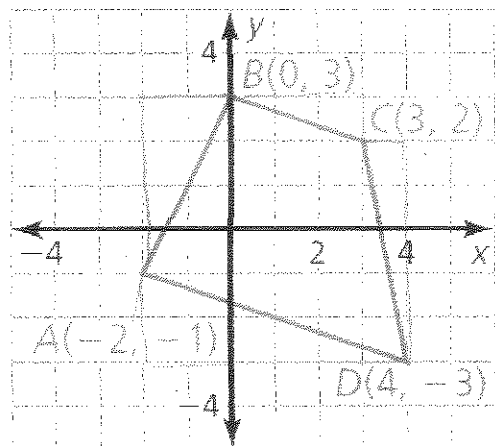
$(BC)^2 = 9 + 1$

$BC = \sqrt{10}$

$(AD)^2 = 2^2 + 6^2$

$(AD)^2 = 4 + 36$

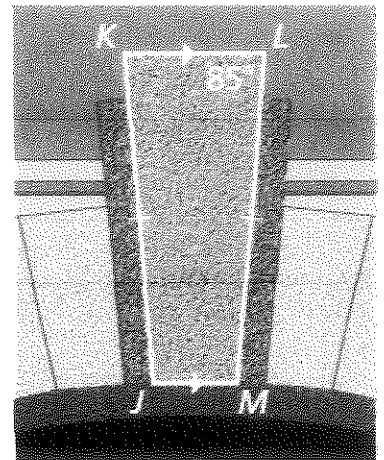
$AD = \sqrt{40}$



**Example 2: Using Properties of Isosceles Trapezoids**

The stone above the arch in the diagram is an isosceles trapezoid.

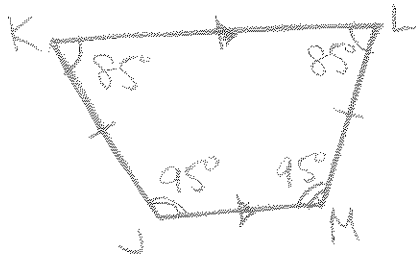
Find  $m\angle K$ ,  $m\angle M$ , and  $m\angle J$ . Explain your reasoning.



$m\angle J = \underline{95^\circ}$

$m\angle K = \underline{85^\circ}$

$m\angle M = \underline{95^\circ}$



Reasoning:

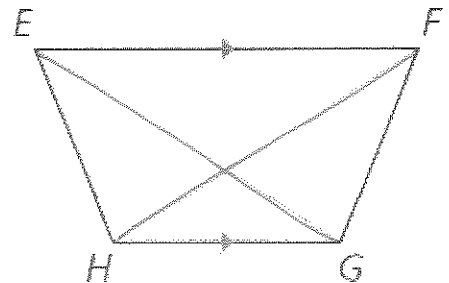
Base angles are congruent

**Example 3:**

If  $EG = FH$ , is trapezoid EFGH isosceles? Yes/No

Explain your reasoning:

If the diagonals are  $\cong$   
then the trapezoid is isosceles



**Example 4:**

If  $m\angle HEF = 70^\circ$  and  $m\angle FGH = 110^\circ$ , is trapezoid EFGH isosceles? Yes/No

Explain your reasoning:

$m\angle FEH + m\angle EHG = 180$

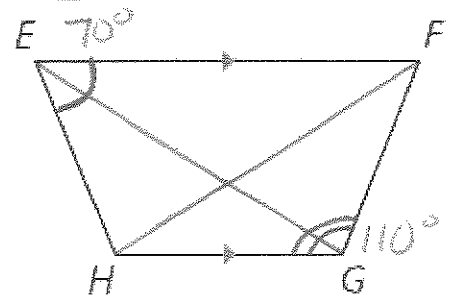
$70^\circ + m\angle EHG = 180$

$m\angle EHG = 110^\circ$

$m\angle EFG + m\angle FGH = 180$

$m\angle EFG + 110 = 180$

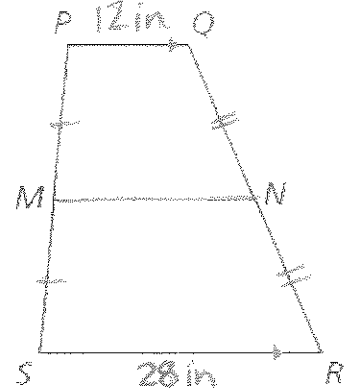
$m\angle EFG = 70^\circ$



$\Rightarrow$  Base angles are congruent.

**Example 5: Using the Midsegment of a Trapezoid**

In the diagram,  $\overline{MN}$  is the midsegment of trapezoid PQRS. Find MN. Show all work.



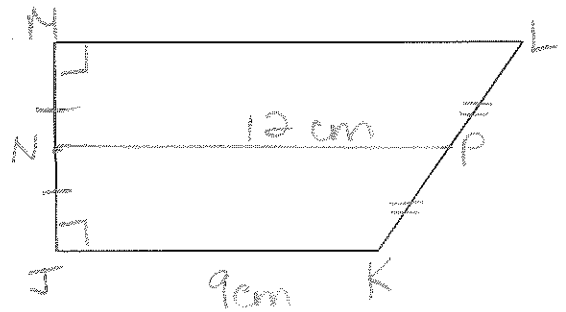
$$MN = \underline{20 \text{ in}} \quad \frac{28 + 12}{2} = \frac{40}{2} = 20$$

**Example 6:**

In trapezoid JKLM,  $\angle J$  and  $\angle M$  are right angles, and JK = 9 centimeters. The length of midsegment NP of trapezoid JKLM is 12 centimeters.

Using a ruler, label trapezoid JKLM and sketch its midsegment. Find ML. Explain your reasoning.

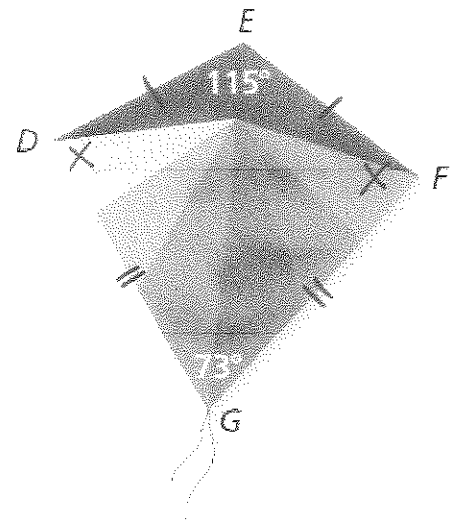
$$ML = \underline{15 \text{ cm}} \quad \begin{array}{ccc} 9 & 12 & 15 \\ & \curvearrowright & \curvearrowright \\ & +3 & +3 \end{array}$$



**Example 7: Finding Angle Measures in a Kite**

Find  $m\angle D$  in the kite shown. Show all work.

$$m\angle D = \underline{86^\circ} \quad \begin{aligned} 115 + 2x + 73 &= 360 \\ 188 + 2x &= 360 \\ 2x &= 172 \\ \boxed{x = 86} \end{aligned}$$



**Example 8:**

In a kite, the measures of the angles are  $3x^\circ$ ,  $75^\circ$ ,  $90^\circ$ , and  $120^\circ$ . Find the value of  $x$ .  
What are the measures of the angles that are congruent? Show all work.

$$x = \underline{25}$$

$$\text{Congruent angle measures} = \underline{75^\circ}$$

$$3x + 75 + 90 + 120 = 360$$

$$3x + 285 = 360$$

$$3x = 75$$

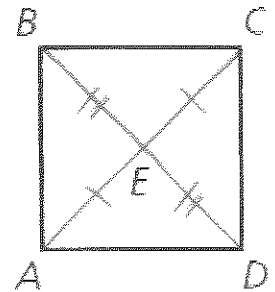
$$\boxed{x = 25}$$

**Example 9: Identifying a Quadrilateral**

What is the most specific name for quadrilateral ABCD?

Explain your reasoning.

Hint: ABCD looks like a square. But you must rely **only** on marked information when you interpret a diagram.



Quadrilateral: Parallelogram

Reasoning: Diagonals bisect each other so quadrilateral is a parallelogram

⇒ No information given about the side lengths or angle measures, so can not determine if it is a rectangle, rhombus or square

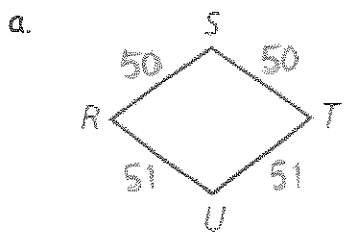
**Example 10:**

Quadrilateral DEFG has at least one pair of opposite sides congruent. What types of quadrilaterals meet this condition?

- Parallelogram
- Rectangle
- Rhombus
- Square
- Isosceles Trapezoid

**Example 11:**

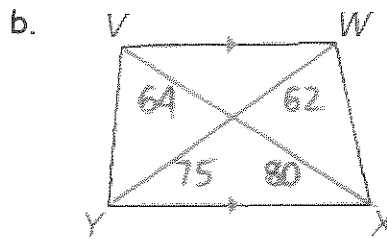
Give the most specific name for each quadrilateral. Explain your reasoning.



Kite

Reasoning:

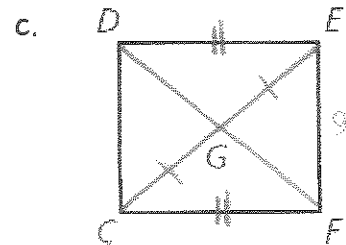
- Two pairs of consecutive sides are congruent
- Opposite sides are not congruent



Trapezoid

Reasoning:

- Two sides are parallel
- Diagonals do not bisect each other



Quadrilateral

Reasoning:

- No enough information to be more specific

